Depositing More Should Not Earn Less

Supplier income monotonicity and rate curve design in pooled lending markets

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Abstract

Utilization dependent interest rate models can create a counterintuitive supplier incentive: adding liquidity may reduce a supplier's total interest income even as the supplier's balance increases. This paper derives the general local condition for supplier income monotonicity. If a supplier represents a fraction α of final pool supply and η is borrow rate elasticity, an additional deposit is income nondecreasing exactly when $\alpha(1+\eta) \leq 1$. Two useful rate curve rules follow from this condition. Protecting every supplier up to a chosen pool share $\bar{\alpha}$ gives the fixed elasticity ceiling $K = (1-\bar{\alpha})/\bar{\alpha}$. Uniformly protecting a fresh depositor from every feasible initial utilization gives the utilization dependent condition $(1-U)R_b'(U) \leq R_b(U)$. For piecewise linear curves, the latter becomes a simple adjacent rate ratio bound. We derive both specializations, distinguish the guarantees they provide, and show how they can be applied to existing interest rate models.

1 Introduction

Pooled lending protocols commonly determine interest rates as a function of utilization. A standard design increases the borrow rate gradually below an optimal utilization level and much more aggressively above it. The steep branch improves incentives to restore liquidity under stress, but it also creates a supplier price impact problem: an additional deposit lowers utilization, which lowers the borrow rate, and the lower rate applies to the supplier's entire balance.

Consequently, a supplier may be better off withholding capital. As a simple illustration, suppose a supplier can place \$10,000 at a 20% annualized supply rate and earn approximately \$2,000 per year. If adding another \$10,000 lowers the applicable supply rate to 8%, the resulting \$20,000 position earns only about \$1,600 per year. The specific numbers are illustrative; the relevant property is that total supplier income need not be monotone in supplied capital.

This paper formalizes that property and derives rate curve constraints that prevent it. The analysis began with Cavey's observation of this failure mode in utilization dependent lending markets. The general result is governed by the supplier's share of the pool and the borrow rate's elasticity with respect to utilization. From it, we develop two distinct design guarantees: one for suppliers below a chosen pool share and another for a fresh depositor entering from any initial utilization.

2 Model

Let

- T denote total supplied tokens after the supplier's transaction,

- B denote total borrowings,
- b denote the supplier's resulting deposited balance,
- $U = B/T$ denote utilization,
- $R_b(U)$ denote the borrow rate at utilization U, and
- $R_s(U)$ denote the supply rate.

Ignoring fees, aggregate borrowers pay approximately $BR_b(U)$ in annualized interest. Distributed across T supplied tokens, the supply rate is therefore

$$R_s(U) = \frac{BR_b(U)}{T} = UR_b(U).$$

The supplier's annualized interest income is

$$I = bR_s(U) = bUR_b(U).$$

Define the supplier's share of final pool supply as

$$\alpha = \frac{b}{T}.$$

The use of final rather than before the transaction share is important: the monotonicity condition applies to the supplier's state after the marginal deposit is added.

Consider an infinitesimal increase $d\delta$ in the same supplier's deposit, holding B fixed during the transaction. Then $db/d\delta = 1$ and $dT/d\delta = 1$, so

$$\frac{dU}{d\delta} = \frac{d}{d\delta}\left(\frac{B}{T}\right) = -\frac{B}{T^2} = -\frac{U}{T}.$$

Thus additional supply reduces utilization locally.

3 Supplier Income Monotonicity

Differentiating $I = bUR_b(U)$ with respect to the additional deposit gives

$$\begin{aligned}
\frac{dI}{d\delta} &= \frac{d}{d\delta}\left[bUR_b(U)\right] \\
&= UR_b(U) + b\left[R_b(U) + UR_b'(U)\right]\frac{dU}{d\delta} \\
&= UR_b(U) - \frac{bU}{T}\left[R_b(U) + UR_b'(U)\right] \\
&= UR_b(U)\left[1 - \frac{b}{T}\left(1 + \frac{UR_b'(U)}{R_b(U)}\right)\right].
\end{aligned}$$

Hence

Proposition 1 (Marginal supplier income)**.** *For a marginal deposit made while borrowings are fixed,*

$$\boxed{\frac{dI}{d\delta} = UR_b(U)\left[1 - \alpha\left(1 + \frac{UR_b'(U)}{R_b(U)}\right)\right].}$$

The derivative contains two competing effects. The marginal dollar earns the current supply rate $UR_b(U)$, while the induced decrease in utilization lowers the rate applied to the supplier's existing balance. The term $UR_b'(U)/R_b(U)$ measures the strength of this rate impact in normalized terms.

3.1 Borrow Rate Elasticity

For a quantity Y that depends on X, elasticity is

$$\varepsilon = \frac{dY/Y}{dX/X} = \frac{X}{Y}\frac{dY}{dX}.$$

Applying this definition to the borrow rate gives

$$\boxed{\eta(U) = \frac{U}{R_b(U)}\frac{dR_b}{dU} = \frac{UR_b'(U)}{R_b(U)}.}$$

Substituting into the income derivative yields the central identity

$$\boxed{\frac{dI}{d\delta} = UR_b(U)\left[1 - \alpha(1 + \eta(U))\right].}$$

Since $U > 0$ and $R_b(U) > 0$, these terms do not affect the sign of the derivative. Therefore

$$\frac{dI}{d\delta} \geq 0 \quad \iff \quad \alpha(1 + \eta) \leq 1.$$

Equivalently,

Corollary 2 (Safe supplier share)**.** *The largest supplier share for which an additional deposit is locally nondecreasing in total interest income is*

$$\boxed{\alpha_{\text{safe}}(U) = \frac{1}{1 + \eta(U)}.}$$

For example, $\eta = 9$ corresponds to a 10% safe share, $\eta = 19$ to 5%, and $\eta = 99$ to 1%. This relation provides a precise meaning for an interest rate curve being "too steep": the relevant object is the percentage sensitivity of the rate to utilization, not the unnormalized slope alone.

4 Fixed Supplier Share Protection

Suppose the design objective is to protect every supplier whose final share of pool supply is at most $\bar{\alpha}$. Requiring $dI/d\delta \geq 0$ for such suppliers implies

$$\bar{\alpha}(1 + \eta) \leq 1,$$

and therefore

$$\eta \leq \frac{1 - \bar{\alpha}}{\bar{\alpha}}.$$

Define

$$\boxed{K = \frac{1 - \bar{\alpha}}{\bar{\alpha}}.}$$

Then the economic policy choice $\bar{\alpha}$ is equivalent to the rate design constraint

$$\boxed{\eta(U) \leq K.}$$

A 5% protection target implies $K = 19$; a 1% target implies $K = 99$. Smaller protected shares permit a more elastic rate curve.

The constant K is not a universal restriction on an interest rate model. It appears only after choosing the policy objective $\bar{\alpha}$: a protocol may decide that every supplier below that share should be protected at every utilization. A different guarantee produces a different constraint, as the next section shows.

5 Fresh Depositor Guarantee

Consider a supplier who begins with no position and deposits x into a pool with initial supply S_0 and fixed borrows B. Let

$$u_0 = \frac{B}{S_0} \qquad \text{and} \qquad u = \frac{B}{S_0 + x}$$

denote utilization before and after the deposit. The supplier's final pool share can be written entirely in terms of these two utilizations:

$$\alpha = \frac{x}{S_0 + x} = 1 - \frac{u}{u_0}.$$

Substituting this expression into the general condition $\alpha(1 + \eta) \leq 1$ gives

$$\left(1 - \frac{u}{u_0}\right)\left(1 + \frac{uR_b'(u)}{R_b(u)}\right) \leq 1.$$

Rearranging yields the exact condition along a deposit path that begins at u_0:

Proposition 3 (Fresh depositor path condition)**.** *For a fresh depositor entering at utilization u_0, total supplier income is nondecreasing at the resulting utilization $u \leq u_0$ if and only if*

$$\boxed{(u_0 - u)R_b'(u) \leq R_b(u).}$$

If the borrow rate is nondecreasing, the most demanding starting point is $u_0 = 1$. A rate curve that protects a fresh depositor beginning from any feasible utilization therefore satisfies

Theorem 4 (Global fresh depositor condition)**.** *For a positive, nondecreasing borrow rate curve, a sufficient condition that is tight in the worst case for fresh depositor income to remain nondecreasing from every initial utilization is*

$$\boxed{(1 - u)R_b'(u) \leq R_b(u).}$$

Equivalently,

$$\boxed{\eta(u) \leq \frac{u}{1 - u}.}$$

Unlike a fixed ceiling K, the permitted elasticity now depends on utilization. The bound is restrictive at low utilization and becomes progressively looser as u approaches one. It therefore permits a steep emergency branch near full utilization while preserving the fresh depositor's income guarantee.

5.1 Piecewise linear form

Consider a nondecreasing linear segment from (u_L, R_L) to (u_H, R_H), with

$$m = \frac{R_H - R_L}{u_H - u_L}.$$

Inside the segment, $R_b(u) = R_L + m(u - u_L)$. The quantity $R_b(u)/(1 - u)$ is increasing when $m \geq 0$, so the tight point is the lower endpoint. The global fresh depositor condition holds throughout the segment exactly when

$$0 \leq m \leq \frac{R_L}{1 - u_L}.$$

Substituting the segment slope gives the adjacent rate rule

$$\boxed{\frac{R_H}{R_L} \leq 1 + \frac{u_H - u_L}{1 - u_L}.} \tag{5.1}$$

If every segment of a continuous piecewise linear curve satisfies Equation 5.1, income remains nondecreasing even when a deposit crosses several segments. No additional condition between segments is required.

Table 1 gives an illustrative curve that becomes sharply responsive near full utilization without violating the condition. A flat clamp after the final knot also preserves monotonicity.

Table 1: A piecewise linear borrow rate curve satisfying deposit monotonicity.

Segment	Rate ratio	Maximum ratio	Result
0% → 80%	1.800	1.800	tight
80% → 90%	1.500	1.500	tight
90% → 95%	1.481	1.500	valid
95% → 98%	1.600	1.600	tight
98% → 99.5%	1.750	1.750	tight
99.5% → 99.99%	1.964	1.980	valid

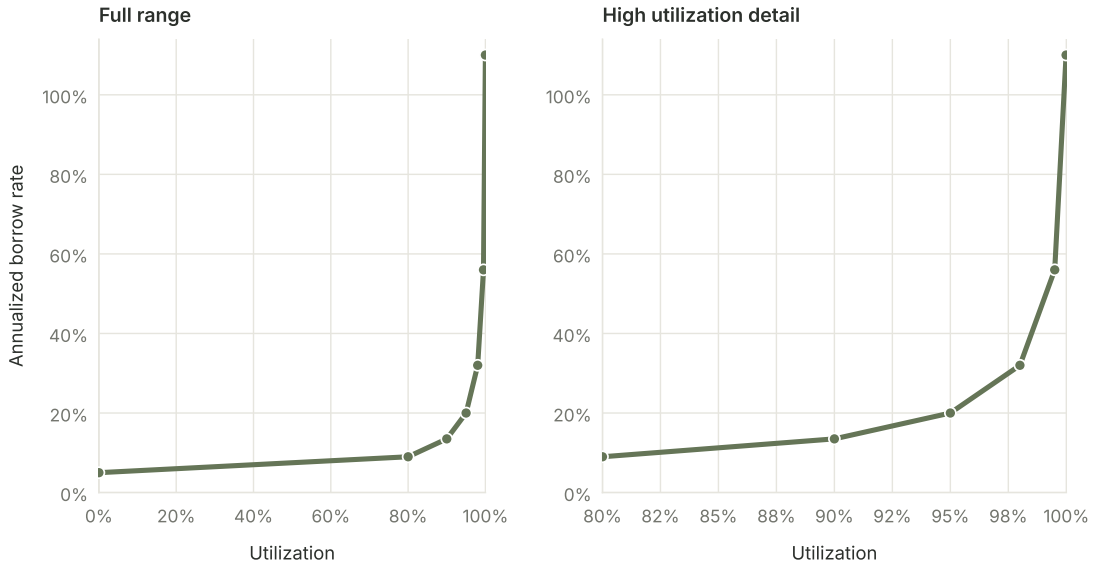

Figure 1: An illustrative piecewise linear curve with knots at 0%, 80%, 90%, 95%, 98%, 99.5%, and 99.99% utilization and corresponding borrow APRs of 5%, 9%, 13.5%, 20%, 32%, 56%, and 110%. Each segment satisfies the adjacent rate rule.

6 The Constant Elasticity Boundary

Consider the standard single kink reference model

$$R_{\text{old}}(U) = \begin{cases} R_o \dfrac{U}{O}, & U \le O, \\ R_o + m(U - O), & U > O, \end{cases}$$

where

$$m = \frac{R_{\max} - R_o}{1 - O}.$$

Below the optimum, $R_b(U)$ is proportional to U, so $\eta = 1$. On the linear branch above the optimum,

$$\eta_{\text{old}}(U) = \frac{Um}{R_o + m(U - O)}.$$

The rate curve need not be changed wherever $\eta_{\text{old}}(U) \le K$. The boundary of the feasible set is obtained by imposing equality, $\eta(U) = K$:

$$\frac{U}{R_b(U)} \frac{dR_b}{dU} = K.$$

Separating variables and integrating,

$$\frac{dR_b}{R_b} = K \frac{dU}{U}, \qquad \ln R_b = K \ln U + C,$$

so

$$R_b(U) = CU^K.$$

If the constrained branch begins at (O, R_o), then $C = R_o/O^K$ and

Proposition 5 (Constant elasticity boundary)**.** *The curve that remains exactly at elasticity K after the kink is*

$$\boxed{R_K(U) = R_o \left(\frac{U}{O} \right)^K.}$$

Its elasticity is identically K on the constrained branch.

This power law is therefore not an arbitrary smoothing function. It is the solution to the differential equation induced directly by the elasticity ceiling.

7 Endpoint Impossibility Bound

The rate model must still reach $R_{\max}$ at full utilization. Fixed endpoints therefore limit how small the worst case elasticity can be.

For any positive differentiable rate curve connecting (O, R_o) to $(1, R_{\max})$,

$$\ln \frac{R_{\max}}{R_o} = \int_O^1 \eta(U)\, d \ln U.$$

If $\eta(U) \le K$ everywhere, then

$$\ln \frac{R_{\max}}{R_o} \le K \ln \frac{1}{O}.$$

Thus the following lower bound is unavoidable.

Theorem 6 (Endpoint impossibility bound)**.** *Any continuous differentiable borrow rate curve satisfying the fixed endpoints* (O, R_o) *and* $(1, R_{\max})$ *must have worst case elasticity at least*

$$\boxed{K^* = \frac{\ln(R_{\max}/R_o)}{\ln(1/O)}.}$$

Equivalently, an elasticity ceiling $K < K^*$ *is infeasible regardless of the curve shape.*

The bound is tight. Setting $K = K^*$ in the constant elasticity curve makes it reach $R_{\max}$ exactly at $U = 1$. Consequently, K^* is the minimum possible worst case elasticity among curves with the same endpoints.

Because protected supplier share is $1/(1 + K)$, the endpoint constraint also yields the largest share that can be guaranteed without moving the endpoints:

Corollary 7 (Maximum protectable share)**.**

$$\boxed{\bar{\alpha}_{\max} = \frac{1}{1 + K^*}.}$$

The optimal utilization, rate at optimal utilization, maximum rate, and supplier protection target therefore cannot be selected independently.

7.1 The 100% supplier edge case

Setting $\alpha = 1$ in the monotonicity condition gives

$$1 + \eta \leq 1 \quad \Longrightarrow \quad \eta \leq 0.$$

A conventional utilization dependent IRM has $R_b'(U) > 0$ and hence positive elasticity, so a supplier that owns the entire pool cannot be protected unless the borrow rate is flat or decreasing with utilization.

The economic reason is immediate. When $b = T$,

$$I = TUR_b(U) = BR_b(U),$$

because $TU = B$. If additional supply is deposited while B remains fixed, utilization and the borrow rate fall. Total borrower interest therefore falls as well, and no other supplier exists to absorb that reduction. Full protection is incompatible with a strictly increasing utilization dependent rate function.

Figures 2 and 3 show the same result for an illustrative pool. The supplier begins as the entire $10 million pool at 95% utilization and adds liquidity while borrows remain fixed. Total interest falls until utilization reaches the flat portion of the rate curve. Its derivative is negative wherever the borrow rate is increasing and reaches zero only where the rate is flat.

8 A Minimal Change Construction

Let U_c denote the first utilization after O at which the constant elasticity boundary catches the original branch above the optimum:

$$R_o \left(\frac{U_c}{O}\right)^K = R_o + m(U_c - O).$$

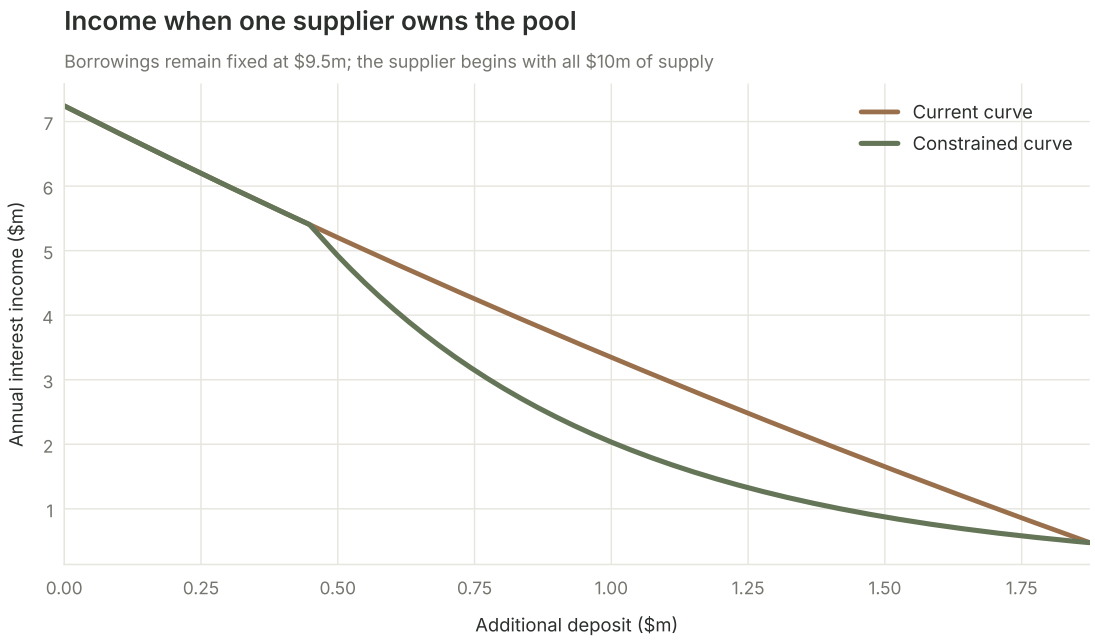

Figure 2: Total annualized supplier income as a 100% supplier adds liquidity to the illustrative pool.

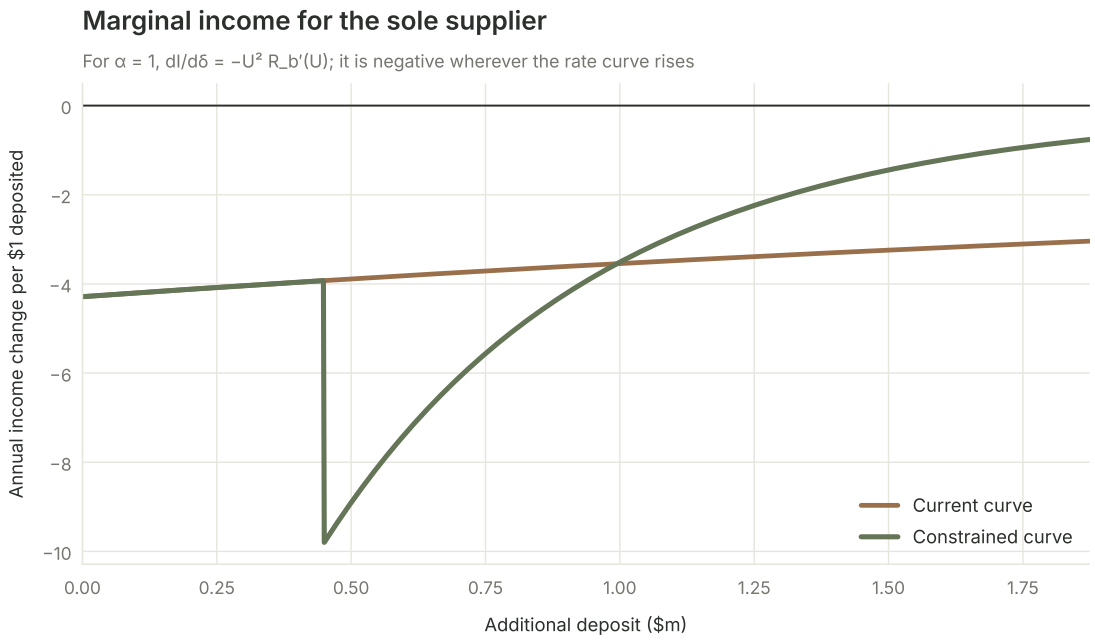

Figure 3: Marginal change in the 100% supplier's annualized income. It is negative wherever the borrow rate curve has positive slope.

For a feasible K, a simple construction is

$$R_{\text{new}}(U) = \begin{cases} R_o \dfrac{U}{O}, & U \le O, \\ R_o \left(\dfrac{U}{O}\right)^K, & O < U < U_c, \\ R_o + m(U - O), & U_c \le U \le 1. \end{cases}$$

Here $m = (R_{\max} - R_o)/(1 - O)$, $K = (1 - \bar{\alpha})/\bar{\alpha}$, and $K \ge K^*$. The construction leaves the original curve unchanged where it already satisfies the elasticity bound, follows the boundary only where necessary, and rejoins the original curve as soon as the constraint permits. If the original curve is feasible everywhere, then no replacement interval is required.

The limiting case is also consistent with the existing model. As $\bar{\alpha} \to 0$, the implied ceiling satisfies $K \to \infty$. Once K exceeds the maximum elasticity of the original curve, the original curve itself becomes feasible and

$$\bar{\alpha} \to 0 \quad \Longrightarrow \quad R_{\text{new}}(U) = R_{\text{old}}(U).$$

Thus the construction can be interpreted as a constraint on the existing IRM rather than a requirement to replace it wholesale.

Figures 4 and 5 compare a standard kinked curve with the minimal change construction for $O = 80\%$, $R_o = 5\%$, $R_{\max} = 100\%$, and $K = 19$. The constrained branch changes the path between the fixed endpoints while retaining both the kink rate and the maximum rate.

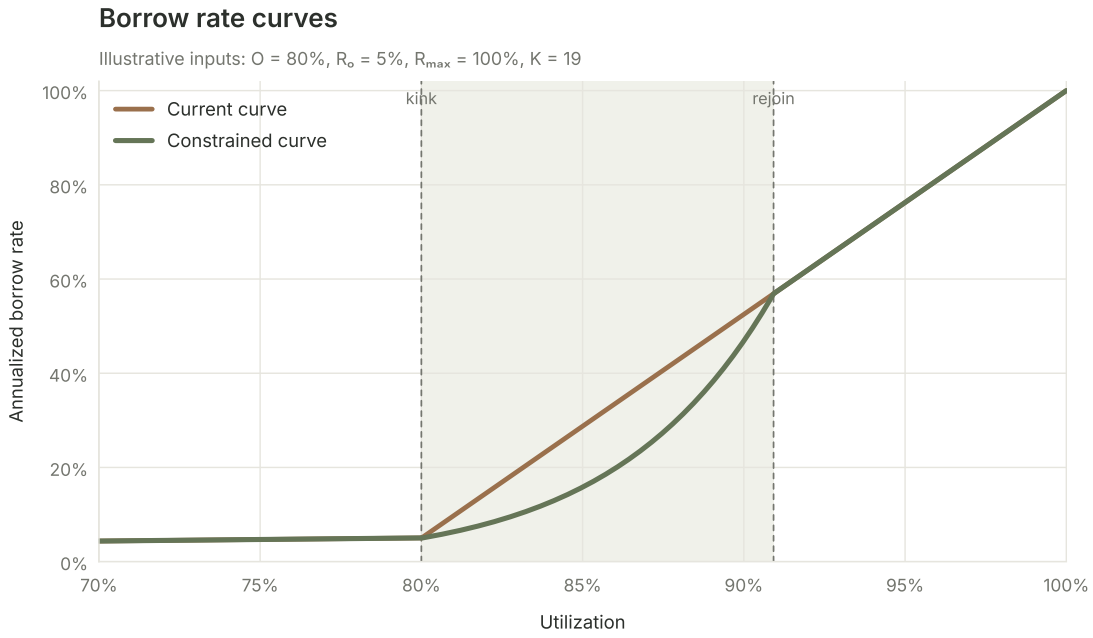

Figure 4: Illustrative borrow rate curves before and after imposing the fixed elasticity ceiling $K = 19$.

For a supplier that begins above the kink and grows to 5% of final supply, the constrained curve keeps both total income and its marginal change nonnegative. The original curve can reduce total income over part of the same deposit path.

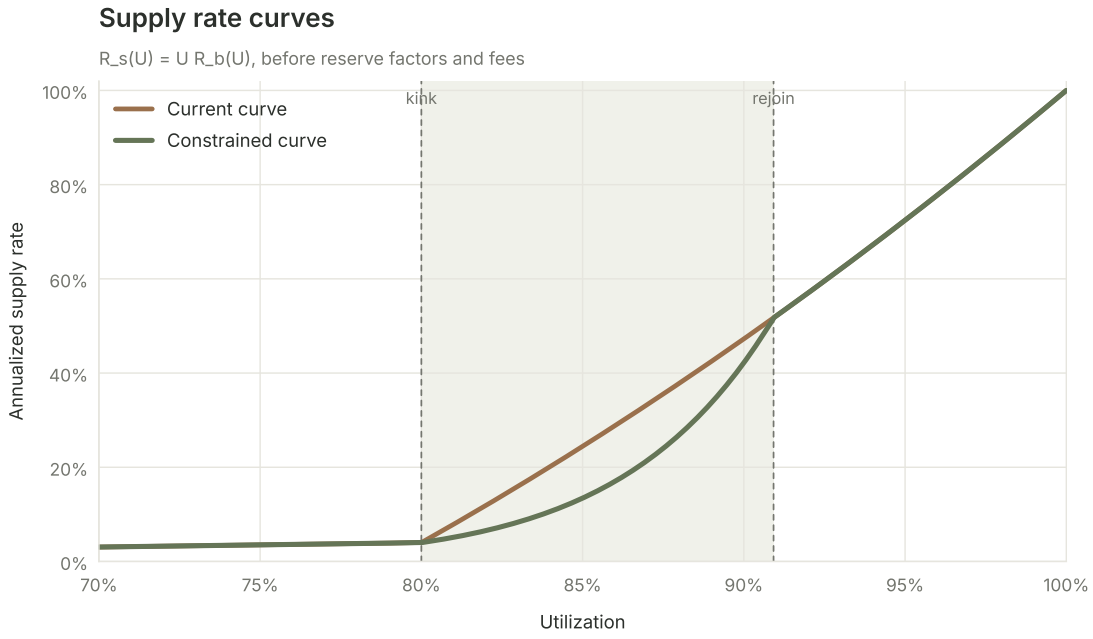

Figure 5: The supply rate curves implied by the borrow rate curves in Figure 4, using $R_s(U) = UR_b(U)$.

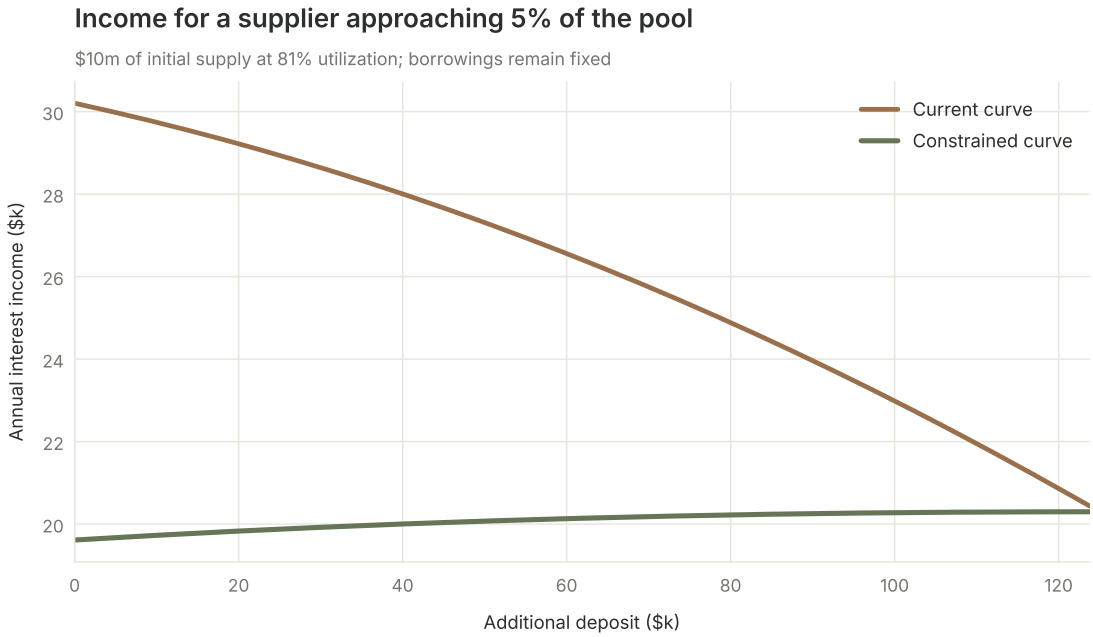

Figure 6: Annualized income along an illustrative deposit path ending at a 5% supplier share.

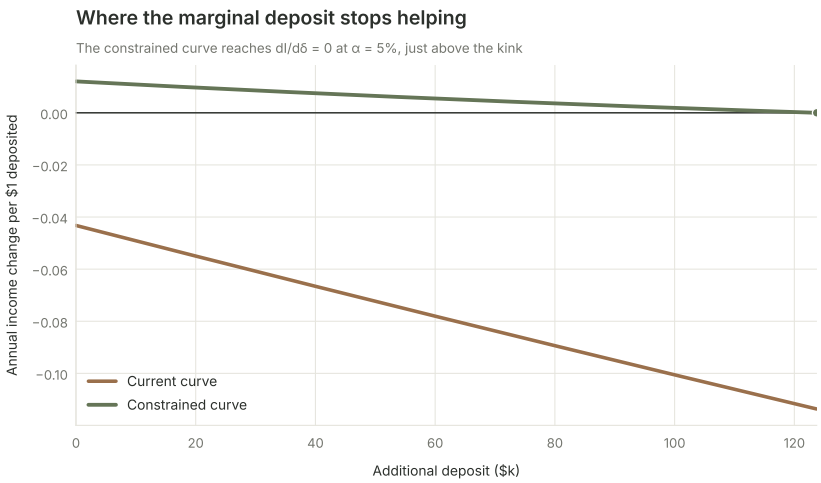

Figure 7: Marginal income along the same path. The constrained curve reaches zero at the protection boundary.

9 Piecewise Linear Fixed Share Check

Onchain implementations commonly represent interest rate curves with linear segments rather than evaluating fractional powers directly. Consider a segment from (u_i, r_i) to (u_{i+1}, r_{i+1}). Its slope is

$$m_i = \frac{r_{i+1} - r_i}{u_{i+1} - u_i},$$

and within the segment

$$R_b(U) = r_i + m_i(U - u_i), \qquad \eta(U) = \frac{U m_i}{R_b(U)}.$$

For a positive linear rate segment, elasticity is monotone within the segment, so enforcing the ceiling at the segment endpoints is sufficient. Writing

$$\Delta r = r_{i+1} - r_i, \qquad \Delta U = u_{i+1} - u_i,$$

the endpoint condition can be expressed without division as

$$\boxed{U\Delta r \le Kr\Delta U.}$$

This form is convenient for fixed point integer arithmetic and allows the elasticity requirement to be imposed directly on an otherwise standard piecewise linear IRM.

10 Conclusion

The general condition $\alpha(1+\eta) \le 1$ separates the supplier's pool share from the rate curve's response to utilization. A fixed supplier share target gives a constant elasticity ceiling K and a boundary curve described by a power law. A fresh depositor guarantee instead gives $(1-u)R_b'(u) \le R_b(u)$ and the adjacent rate ratio rule in Equation 5.1. These are different protections derived from the same condition. Both can be checked against an existing IRM, and the appropriate one depends on which supplier path the protocol intends to make monotone.